The Fading Memory 4th Order Polynomial

The Fading Memory Polynomial Fit.

In a previous article the least Squares technique was used to fit a 4th order polynomial through 60 closing 5 minute price bars of the S&P 500 futures to create a curve that served as a proxy for the market trend. When the curve moved up by a certain percent from its previous local low, the trend was assumed to have changed to the upside and a buy signal was given. When the curve moved down by a certain percent from its previous local high, the trend was assumed to have changed to the downside and a sell signal was given.

Here a different but related technique called the *Fading Memory Polynomial* will be used. This is a mathematical technique that fits a 4th order polynomial to the last ***n*** price bars but calculates the n coefficients of the polynomial such that the error between the polynomial and the current bar is weighted much higher than the error between the price n bars ago and the value of the polynomial n bars ago. As an example, if the latest price is at time t and the price made a turn at time bar t-10 , then we do not want prices prior to t-10 effecting the polynomial fit as much. As will be shown the most familiar case of this fading memory technique is the exponential moving average. The fading memory technique is in contrast to the Least Squares Polynomial fit, which weights all past errors between the polynomial and the price bar equally.

Consider a time series x(t) where t is an integer value (a price bar number) like the number of days or minutes, etc from some starting time. Suppose we want to find at some given time some *n*th-degree polynomial that fits the data well at current and recent prices but ignores the fit as we move into the distant past. One way to construct this type of fit would be to weight the past data with a number that got smaller and smaller the further back in time we went. If we let the polynomial function be represented by the symbol $p(t-\tau)$ where $p(t-0)$ is the current value of the polynomial, $p(t-1)$ is the previous value of the polynomial, etc., then an error function can be formed that consists of the weighted sum of the squared difference between the price series $x(t-\tau)$ and the polynomial $p(t-\tau)$ given by

$$\mathbf{error} = \sum \beta^{\tau}(x(t-\tau) - p(t-\tau))^2 \quad \tau=0 \text{ to } \infty \qquad (1)$$

where $\mathbf{0 < \beta < 1}$ **and** β^{τ} is much less than **1** for large τ.

It turns out that if we let the nth degree polynomial $p(t-\tau)$ be constructed as a linear combination of orthogonal polynomials called Meixner polynomials(please see References 1,2 and 3 for details) then minimizing the error with respect to the coefficients of the orthogonal polynomials yields the best estimate of $\mathbf{x(t-\tau)}$ as $\mathbf{x_{est}(t-\tau)}$ and given by the equation

$$x_{est}(t-\tau) = (1-\beta)\sum_{k=0}^{n} \beta^k b_k(t)\Phi_k(\tau) \qquad (2)$$

Where

$$\Phi_n(t) = \sum_{k=0}^{n} \binom{n}{k}\binom{t}{k} z^k$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$b_j(t) = \sum_{k=0}^{\infty} \beta^k \Phi_j(k) x(t-k)$$

$$\mathbf{z = 1 - 1/\beta}$$

where n is the polynomial degree, $\mathbf{\Phi_k(\tau)}$ are the Meixner polynomials of degree k, and $\mathbf{b_k(t)}$ are the coefficients that minimize the error of equation (1). Generally the summation for $b_j(t)$ can be terminated when β^k is less then 0.01.

For the exact mathematical solutions that produce equation (2) and the mathematical descriptions of the Meixner polynomials refer to References 1,2 and 3.

To yield the 1 day ahead prediction the above equation becomes;

$$\mathbf{x_{est}(t+1)=(1-\beta)\sum\beta^k b_k(t)\Phi_k(-1) \quad k=0 \text{ to } n} \qquad \mathbf{(3)}$$

One case is of immediate interest where the polynomial is a constant, that is ***n***=0.

For this case the solution to equation (3) can be found after some algebraic manipulation to be:

$$\mathbf{X0_{est}=\beta*X0_{est}[1] +(1-\beta)*x(t)} \qquad \mathbf{(4)}$$

Where $\mathbf{X0_{est}[1]}$ is the previous estimated value, x(t) is the current bar’s price and where the 0 in $\mathbf{X0_{est}}$ indicates that we are estimating a polynomial of degree 0 or simply a constant. If a change of variables is made letting $\mathbf{\alpha=(1-\beta)}$ then equation (4) becomes:

$$\mathbf{X0_{est}=(1-\alpha)*X0_{est}[1] +\alpha*x(t)} \qquad \mathbf{(5)}$$

This is the familiar formula for the exponential moving average.

Higher orders of ***n*** don't yield such compact solutions as the case where ***n***=0 .

Fading Memory 4th Order Polynomial System Defined.

The fading memory 4th order polynomial best estimate of the next bars value, $\mathbf{X4_{est}(t+1)}$**,** is constructed at each bar by solving equation 3 with ***n***=4.. The $\mathbf{X4_{est}(t+1)}$ value is then plotted under the price chart. In general what we will be doing is following the plotted curve of $\mathbf{X4_{est}(t+1)}$. When the curve increases by a percentage amount *pctup* from the previous prior low of the curve we will go long. When the curve falls by the percentage amount *pctdn* from the previous prior high of the curve we will go short. For this article 5 minute bars of the S&P Mar/00 futures contract will be used for the price series.

Buy Rule:

- **IF** $\mathbf{X4_{est}(t+1)}$ has moved up by more than the percentage amount of *pctup* from the lowest low recorded in $\mathbf{X4_{est}(t+1)}$ while short then buy one S&P Mar/00 contract at the market.

Sell Rule:

- **IF** $\mathbf{X4_{est}(t+1)}$ has moved down by more than the percentage amount *pctdn* from the highest high recorded in $\mathbf{X4_{est}(t+1)}$ while long then sell one S&P Mar/00 contract at the market.
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Exit Rule

Exit all trades at close of each day with a market on close order.

Walk Forward Optimization

Walk forward optimization will be used here because of the changing nature of the intraday S&P futures market. Intraday price dynamics are constantly changing due to current economic surprises, events and trader sentiment. Also the time of year changes the nature of intraday markets, such as the seasons, holidays, vacation time, etc. As such, optimizations on intraday data performed 3 months ago may no longer be representative of today's intraday price dynamics.

The walk forward procedure will be applied as follows. A period of 22 trading days of the S&P Mar/00 5 minute bar data from January 11th, 2000 through February 11th, 2000, is chosen and system parameter values are found through optimization on this intraday data segment. The parameter values found in the test segment are then applied to the out-of-sample 5 minute intraday bar data following the test segment which in this case is February 14th, 1999 to February 18th, 2000.

Why a 22 trading days for the intraday data test segment? Why not 10 trading days or 44 days? Well the answer is that there is no correct ratio of test data needed to produce good one week intraday out-of-sample results. By experimenting with different window lengths, the four to one ratio seemed to work well. In walk forward testing, enough data is needed to model most of the price dynamics that will be encountered in the out-of-sample segment, but not so much data that when the price dynamics start to change they are swamped by the weight of distant past data price dynamics that no longer are valid. An important unspoken point in walk forward testing is that if you can not get good results in the out-of-sample segments, then the price dynamics cannot be modeled with the system . This means that real time performance will be random using the model. Traders observe this type of random performance (that is it looks great on paper but falls apart in real time) when trying systems based on curve fitting or anecdotal "proof" (looking at 3 or 4 successful cases only) without any out-of-sample testing .

Finding The System Parameters Using Walk Forward Optimization
There are three system parameters to find β, *pctup, and pctdn*.

The best parameters will be defined as those values that give the best Net Profits with the maximum winning bars, minimum losing bars, minimum drawdown, minimum largest losing trades. In addition, the results should be stable, e.g. the profits, wins, and drawdowns should not change by much as the parameters move by a small amount away from their optimum values. Also in choosing the "best" parameters, only those parameters sets whose maximum consecutive losses were 4 or less were considered. Optimization is defined as the search for the parameter values that give the best results as defined above. It should be noted that in this stage of system development, the only thing indicated by the optimum values that are found in the test portion is that the data has been *curve fitted* as best it can with this system. Without further testing on out-of-sample data there is no way to tell if the system will work in the future.

It is not well known, but almost any real time series or even a random time series defined over a fixed number of bars can be curve fitted rather easily. The performance results and the statistical measurements that validate this performance of the curve fit will look excellent giving the false illusion of future profitability. However, the truth is that these excellent performance and associated statistics on the test section in no way validate how the system will perform on data it has not been optimized on. Only out-of-sample testing, that is testing on data the parameters were not derived on, can determine if the parameters found in the test section have captured the price dynamics. For instance in the Fading Memory polynomial fit process the error minimization forces the generated curve to fit the past data like a glove. It's almost impossible not to get an excellent fit with excellent statistical results. Unfortunately, this excellent fit in no way implies that the system will perform equally well on out-of-sample data, it just tells us we have a very good curve fit.

Results
Figure 1 presents a table of the test window optimum parameters for the Fading Memory Polynomial system using the S&P Mar/00 5min bar data series.

Start Date	End Date	β	Pctup	Pctdn
1/11/2000	2/11/2000	0.910	1.10%	0.60%

Figure 1 Optimum Parameter Values For Test Data Segment

Figure 2 presents the performance summary using the optimum parameters for the test segment shown in Figure 1.

Figure 3 presents the performance summary of the out-of-sample data segment from 2/14/2000 to 2/18/2000. This performance represents what would have happened in *real time* if one used the parameters found in the test section. Slippage, and commissions are not included.

Figure 4 presents a trade by trade summary from 1/14/2000 to 2/18/2000. Note only the out of sample trades are presented here for the in-sample trades of the test section were generated by the curve fit.

Figures 5A and 5B present the 5 minute bar charts of the S&P Mar/00 futures with the 4^{th} order Fading Memory Polynomial Curve and all the buy and sell signals from the trade by trade summary of Figure 4 indicated on the charts. Also on these charts the exponential average with $\boldsymbol{\alpha=(1-\beta)}$ is included for comparison. The Fading Memory curve is colored red while the exponential average curve is colored black.

Discussion of System Performance

As can be observed from the test sample Performance summary in Figure 2 and the out-of-sample performance summary of Figure 3, the out-of-sample performance was comparable to the test sample performance. This comparable performance indicates that 22 days in the test section was enough to capture the price dynamics so that the system would perform well in the out-of-sample section. This is not always the case. Many times the underlying price dynamics change abruptly creating loses in the out-of-sample section. However if the test window slides forward every week then the new price dynamics are quickly captured and the out-of-sample profits should return.

Observing the out-of-sample trade by trade summary of Figure 4, we can see that the system did equally well on longs and shorts. This is a good sign showing that neither longs nor shorts were favored in the current market. Maximum trade drawdowns were very low for the S&P50 Mar/00 contract. The low drawdowns were probably due to the down trending nature of the S&P's during the out-of-sample section. From Figures 2 and 3 the average trade (win & loss) was $2995 in the test section and $3165 in the out-of-sample section. Beside being impressive, the almost equal average trade results also indicates stability in the parameter selection.

In examining the charts we can see that the Fading Memory Polynomial curve did a very good job in smoothing the price series while not lagging. The curve had anywhere from a zero bar lag to a 2 bar lag from the major tops and bottoms.

As good as this system looks, please be aware that in order to use this system in real time trading, at least ten to twenty more test and out-of-sample windows from the past would have to be examined to gain confidence that the results above were not due to pure chance.

References:

1. A. Erdelyi et al, *Higher Transcendental Functions*. New York: McGraw-Hill, 1953.
2. Abramowitz and Stegun, Ed., Handbook of Mathematical Functions, New York: Dover, 1972

Info on Dennis Meyers

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Figure 2 Test Window Performance Summary for S&P Mar/00
Fading Memory Polynomial System 01/11/2000 - 02/11/2000

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!FadingMemPoly  S&P Mar/00-5 min   01/11/2000 - 02/11/2000

       Performance Summary:  All Trades

Total net profit      $ 42000.000    Open position P/L     $     0.000
Gross profit          $ 62900.000    Gross loss            $-20900.000

Total # of trades          38        Percent profitable         55%
Number winning trades      21        Number losing trades       17

Largest winning trade $ 13325.000    Largest losing trade  $ -3075.000
Average winning trade $  2995.238    Average losing trade  $ -1229.412
Ratio avg win/avg loss      2.436    Avg trade(win & loss) $  1105.263

Max consec. winners         4        Max consec. losers          3
Avg # bars in winners      46        Avg # bars in losers       41

Max intraday drawdown $ -5025.000
Profit factor               3.010    Max # contracts held        1
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Figure 3 Out-Of-Sample Performance Summary for S&P Mar/00
Fading Memory Polynomial System 02/14/2000 - 02/18/2000

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!FadingMemPoly  S&P Mar/00-5 min   02/14/2000 - 02/18/2000

       Performance Summary:  All Trades

Total net profit      $ 11925.000    Open position P/L     $     0.000
Gross profit          $ 15825.000    Gross loss            $ -3900.000

Total # of trades           8        Percent profitable         63%
Number winning trades       5        Number losing trades        3

Largest winning trade $  8550.000    Largest losing trade  $ -1925.000
Average winning trade $  3165.000    Average losing trade  $ -1300.000
Ratio avg win/avg loss      2.435    Avg trade(win & loss) $  1490.625

Max consec. winners         3        Max consec. losers          2
Avg # bars in winners      48        Avg # bars in losers       51

Max intraday drawdown $ -2800.000
Profit factor               4.058    Max # contracts held        1
Account size required $  2800.000    Return on account         426%
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FIGURE 4 Out-Of-Sample Trade By Trade Summary
S&P Mar/00 5min FadingMemPoly System 02/14/2000 - 02/18/2000

Entry Date	Entry Time		Entry Price	Exit Date	Exit Time	Exit Price	Bars InTrd	Trade $P&L	Trade %P&L	Trade Max$Pft	Time	Trade Max$DD	Time
02/14/2000	940	Sell	1398.5	02/14/2000	1615	1399.0	79	($125)	-0.04%	$2,825	1335	($375)	1155
02/15/2000	940	Sell	1396.1	02/15/2000	1455	1403.5	63	($1,850)	-0.53%	$3,275	1130	($1,975)	1450
02/15/2000	1455	Buy	1403.5	02/15/2000	1615	1410.0	16	$1,625	0.46%	$2,375	1540	$0	1455
02/16/2000	940	Sell	1404.0	02/16/2000	1615	1392.0	79	$3,000	0.85%	$3,000	1615	($1,500)	1005
02/17/2000	940	Sell	1403.8	02/17/2000	1340	1402.0	48	$450	0.13%	$4,700	1200	($50)	940
02/17/2000	1340	Buy	1402.0	02/17/2000	1435	1394.3	11	($1,925)	-0.55%	$0	1340	($2,050)	1420
02/17/2000	1435	Sell	1394.3	02/17/2000	1615	1385.5	20	$2,200	0.63%	$2,200	1615	($625)	1435
02/18/2000	940	Sell	1387.5	02/18/2000	1615	1353.3	79	$8,550	2.46%	$9,375	1545	$0	940

FIGURE 5A S&P Mar/00 5min FadingMemPoly System 02/14/2000 - 02/18/2000

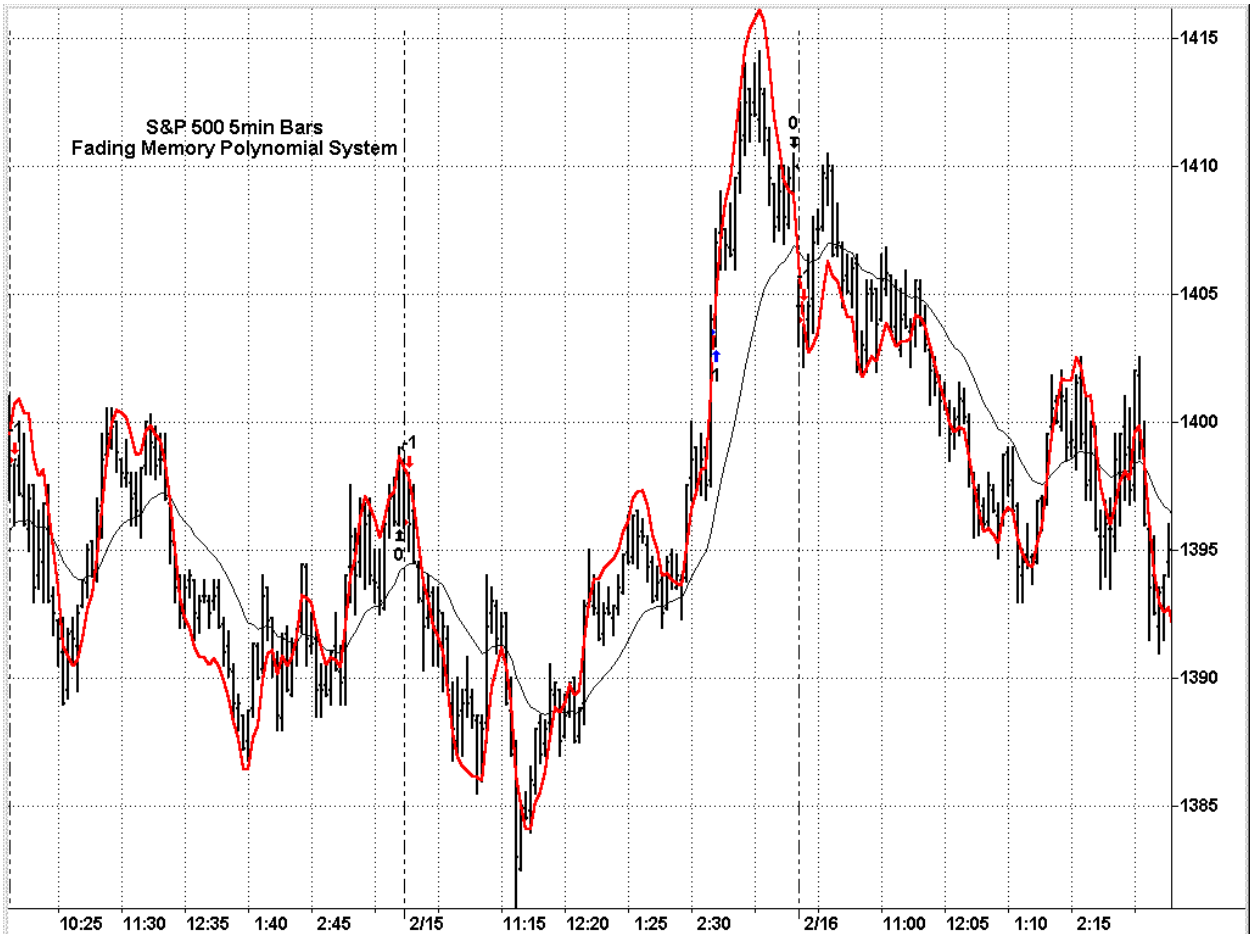

FIGURE 5B S&P Mar/00 5min FadingMemPoly System 02/14/2000 - 02/18/2000